SYLLABUS STRUCTURE
FOR
DOCTOR OF PHILOSOPHY(Ph.D.) PROGRAMME
IN
MATHEMATICS

(Ph.D. in Mathematics)
(Academic Year 2025-26)

2025

SCHOOL OF MATHEMATICS
GANGADHAR MEHER UNIVERSITY, AMRUTA VIHAR

SAMBALPUR, ODISHA-768004, INDIA

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VISION

To foster in mathematical education and research, technical excellence, well poised between abstraction and application.

MISSION

M-1-To offer globally-relevant, research-focused, technology-enabled education at the graduate, post-graduate and research levels in various areas of Mathematical sciences.

M-2-To contribute to the development of students as mathematical thinkers, enabling them to become lifelong learners, to continue to grow in their chosen professions, and to function as productive citizens.

M-3-To develop and conduct continuing education programs with a view to update their fundamental knowledge base and problem-solving capabilities in the various areas of core specialization of the University.

M-4-To develop comprehensive linkages with premier academic and research institutions within the country and abroad for mutual benefit.

PROGRAMME OUTCOMES
(Chosen from the allowed list as set by UGC)

The programme outcomes and attributes are attained by the postgraduate students of Gangadhar Meher University through learning acquired on completion of a programme of study. Individual programmes of study have a defined programme specific learning outcomes which needs to be attained for the award of a specific degree. The programme learning outcomes of Gangadhar Meher University focus on various aspects of knowledge and skills that prepare students for further study, employment, and citizenship. Therefore, the PG programme of this University has been designed with the objective to develop in-depth knowledge of students in frontier areas of concerned subject and seeks to achieve the following:

PO-1: Knowledge and Understanding:
Capable of demonstrating comprehensive knowledge and understanding of one or more disciplines that form a part of a post-graduate programme of study.

PO-2: General, Technical and Professional Skills:
Enable students to enhance mathematical skills and understand the fundamental concepts of pure and applied mathematics.

PO-3: Application of Knowledge and Skills:
Capability to apply analytic thought to a body of knowledge; analyze and evaluate evidence, arguments, claims, beliefs on the basis of empirical evidence; identify relevant assumptions or

implications; formulate coherent arguments; critically evaluate practices, policies and theories by following scientific approach to knowledge development.

PO-4: Research Skills:
A sense of inquiry and capability for asking relevant/appropriate questions, problematizing, synthesizing and articulating; Ability to recognize cause-and-effect relationships, define problems, formulate hypotheses, test hypotheses, analyze, interpret and draw conclusions from data, establish hypotheses, predict cause-and-effect relationships; ability to plan, execute and report the results of an experiment or investigation.

PO-5: Generic Learning Outcomes:
Capable of strengthening knowledge and understanding in deeper stage and developing the art of skills. Along with this, constructing a good moral and ethics in the students.

PO-6: Constitutional, Humanistic, Ethical and moral values
Ability to embrace moral/ethical values in conducting one's life, formulate a position/argument about an ethical issue from multiple perspectives, and use ethical practices in all work. Capable of demonstrating the ability to identify ethical issues related to one's work, avoid unethical behavior such as fabrication, falsification or misrepresentation of data or committing plagiarism, not adhering to intellectual property rights; appreciating environmental and sustainability issues; and adopting objective, unbiased and truthful actions in all aspects of work.

PO-7: Employability and Job skills, Entrepreneur skills
Postgraduate study boosts the career progress and chart out the career paths. It demonstrates the ability to tackle complex and challenging assessment tasks.

PROGRAMME SPECIFIC LEARNING OUTCOME (PSO)

(Set by School of Mathematics)

PSO-1: Will have a strong foundation in both pure and applied mathematics.

PSO-2: Will be able to apply mathematical skills for solving problems and for preparing various competitive exams.

PSO-3: Will be able to communicate mathematical knowledge effectively, in writing as well as orally.

PSO-4: Will identify applications of mathematics in other disciplines, leading to enhancement of career prospects in different fields and research areas.

PSO-5: Will have basic knowledge of programming and computational techniques as required for employment.

PSO-6: Should have the knowledge of the fundamental axioms in mathematics and capability of developing ideas based on them and inculcate mathematical reasoning.

PSO-7: Will be able to locate and analyze the different mathematical texts with appropriate theoretical framework.

PSO-8: Have the knowledge of a wide range of mathematical techniques and application of mathematical methods/tools in science, social science, engineering and technology.

Matching

Matching Percentage	Level Indicator
> 70%	3
= 60 %	2
< 50%	1

MISSION TO PO MAPPING

	PO-1	PO-2	PO-3	PO-4	PO-5	PO-6	PO-7
M1	3	3	3	3	3	3	3
M2	3	2	3	3	3	2	3
M3	3	2	2	3	2	2	2
M4	1	2	1	3	1	1	2

PSO TO PO MAPPING

	PO-1	PO-2	PO-3	PO-4	PO-5	PO-6	PO-7
PSO1	3	3	2	3	3	3	3
PSO2	1	3	3	2	3	3	3
PSO3	3	2	2	2	2	3	2
PSO4	1	2	3	3	3	2	2
PSO5	2	3	3	2	3	2	2
PSO6	1	2	2	3	1	1	3
PSO7	2	3	2	3	1	3	2
PSO8	1	2	3	3	3	2	3

Brief Overview of Syllabus

Semester	Subject Code	Subject Name	Credit	Mid Sem	End Sem	Total Marks
I	Paper – 711A	Advanced Functional Analysis	4	20	80	100
	Paper – 711B	Advanced Number Theory	4	20	80	100
	Paper – 711C	Fractional Calculus	4	20	80	100
	Paper - 712	Research and Publication Ethics	2	20	80	100
	Paper - 713	Research Methodology	4	20	00	100
	Paper - 714	Dissertation or Review Writing/Presentation/ Viva-Voce	8			200 (150+25 +25)
		Total	**18**			**500**

N.B. -Ph.D. Scholars have to choose one paper from 711A, 711B and 711C**.**

SEMESTER – I

Paper – 711A: ADVANCED FUNCTIONAL ANALYSIS (4L-0T-0P)

Full Marks: 100 (20 Mid Term + 80 End Term)

Pre-requisites:
Functional Analysis, Operator Theory

Objective:

Syllabus-:

Unit-I **[12 Hours]**

Spectrum of a bounded operator: point spectrum, continuous spectrum and residual spectrum, spectral properties of bounded linear operators, resolvent and spectrum, spectral mapping theorem for polynomials. Non-emptiness of the spectrum of a bounded linear operator on a complex Banach space, spectral radius, spectral radius formula, Banach algebras, resolvent set and spectrum of a Banach algebra element, further properties of Banach algebras, spectral radius of a Banach algebra element, non-emptiness of the spectrum of a Banach algebra element.

Unit-II **[12 Hours]**

Compact linear operators on normed spaces, compactness criterion, weak convergence and compact operators, separability of range, adjoint of compact operators, Spectral properties of compact linear operators on normed spaces, eigen values of compact linear operators, closedness of the range of T , spectral properties of compact linear operators, Operator equations involving compact linear operators, necessary and sufficient conditions for the solvability of various operator equations, Fredholm type theorems, Fredholm alternative Theorem.

Unit-III **[12 Hours]**

Spectral theory of bounded self-adjoint linear operators: spectral properties of bounded self-adjoint operators, positive operators, projection operators and their properties. Spectral family of a bounded self-adjoint linear operator, spectral representation of bounded self-adjoint linear operators, spectral theorem for bounded self-adjoint linear operators, extension of the spectral theorem to continuous functions, properties of the spectral family of a bounded self-adjoint operator.

Unit-IV **[12 Hours]**

Unbounded linear operators and their Hilbert adjoints, Hellinger-Toeplitz theorem, Hilbert-adjoint, symmetric and self-adjoint linear operators. Closed linear operators and closures, spectral properties of self-adjoint linear operators. Spectral representation of unitary operators : Weckens lemma, spectral theorem for unitary operators, spectral representation for self-adjoint linear operators, multiplication and differentiation operators.

References:

1. E. Kreyszig, Introductory Functional Analysis with Applications, Wiley Publisher
2. M. T. Nair, Functional Analysis: A First Course, PHI Publsiher, Second Edition

Course Outcomes:

Upon successful completion of this course students able to-:

CO 1: Spectral Theory

CO 2: Spectral Theory of compact operators

CO 3: Spectral theory of bounded operators

CO 4 Spectral theory of unbounded operators

CO 5: spectral representation for self-adjoint linear operators

CO-PO Mapping (Paper 711A)

	PO-1	PO-2	PO-3	PO-4	PO-5	PO-6	PO-7
CO1	3	2	1	1	2	1	1
CO2	3	1	3	2	2	1	2
CO3	2	3	3	3	2	2	2
CO4	2	2	3	2	3	2	3
CO5	2	3	2	3	2	2	2

Programme articulation matrix row for Paper 711A

	PO-1	PO-2	PO-3	PO-4	PO-5	PO-6	PO-7
Course Paper **711A**	3	2	3	2	3	2	3

CO-PSO Mapping (Paper 711A)

	PSO-1	PSO-2	PSO-3	PSO-4	PSO-5	PSO-6	PSO-7	PSO-8
CO1	3	3	3	3	3	3	2	2
CO2	3	2	3	2	2	2	3	3
CO3	3	3	3	3	2	2	3	3
CO4	2	3	3	2	3	3	2	2
CO5	2	3	2	3	3	2	3	2

Paper – 711B: ADVANED NUMBER THEORY(4L-0T-0P)

Full Marks: 100 (20 Mid Term + 80 End Term)

Pre-requisites:
Elementary Number Theory, Complex Analysis, Continued Fractions, Congruences**,**

Objective:
The objective of the course is to introduce theory related to Ramanujan's work, viz, q-series, theta functions, Ramanujan's continued fractions, hypergeometric series. Furthermore, the main objective of is to introduce partition theory and its congruences. Finally, to introduce on modular form of different level.

Syllabus-:

Unit-I **[12 Hours]**

q-series, theta functions, Fundamental Theorems about q-Series and Theta Functions Definition of hypergeometric series and elliptic integrals, The main theorem Application of main theorem, Sum of square and triangular numbers, modular equations
The Rogers-Ramanujan Continued Fractions, Convergence, Divergence and Value of R(q), other q-continued fractions, Identities for R(q), Modular Equations for R(q).

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Unit-II **[12 Hours]**

Introduction to partitions, geometric representation, generating functions, Congruences for p(n) and $\tau(n)$

Unit-III **[12 Hours]**

Modular Group, The upper half plane, Modular form of level one, The Ramanujan's τ function, Modular forms of higher level

Unit-IV **[12 Hours]**

The Petersson inner product, Hecke operators on higher level, Dirichlet series and modular forms, Special topics.

Prescribed Book

1. Bruce C. Berndt, Number Theory in the Spirit of Ramanujan, American Mathematical Society, 2006. Chapter 1(1.1-1.3), 2(2.1-2.5), 5(5.1-5.4), 6(6.1-6.3), 7(7.1-7.5).
2. George. E. Andrews, The Theory of Partition, Cambridge University press, 1984. Chapter 1(1.1-1.3).
2. M. Ram Murty, M. Dewar, and H. Graves, Problems in the Theory of Modular Forms, Hindustan Book Agency, 2015. Chapter 2(2.1-2.4),3 (3.1-3.3), 4(4.1-4.5), 5(5.1-5.4), 6(6.1-6.5), 7(7.1-7.4), 8(8.1-8.3), 9(9.1-9.4), 10(10.1-10.4)

Reference Books

1. Andrews, G.E. *Number Theory*, Hindustan Publishing Corporation, New Delhi, 1992).
2. Bruce C. Berndt – Ramanujan's Note Books Volume-1 to 5, Springer.
3. G. E. Andrews – Number Theory, Dover Books,1995.
4. N. Koblitz, Introduction to Elliptic Curves and Modular Forms, GTM, vol. 97, Springer, 1993.
5. J. P. Serre, A Course in Arithmetic, GTM, vol. 7, Springer, 1973.

Course Outcomes:

CO 1: Understand the concept of q-series, continued fractions, hypergeometric function and theta functions specialization to Ramanujan's work

CO 2: Study the basic concepts on partition theory and their congruence properties

CO 3: Study the concepts of modular group and modular form

CO 4: Enhance the concept on modular form in higher level

CO-PO Mapping (Paper 711B)

	PO-1	PO-2	PO-3	PO-4	PO-5	PO-6	PO-7
CO1	2	3	3	2	1	1	1
CO2	3	1	2	3	2	3	2
CO3	2	3	3	3	3	2	3
CO4	2	3	3	2	3	2	3

Programme articulation matrix row for Paper 711B

	PO-1	PO-2	PO-3	PO-4	PO-5	PO-6	PO-7
Course Paper 711B	2	3	3	3	2	3	3

CO-PSO Mapping (Paper 711B)

	PSO-1	PSO-2	PSO-3	PSO-4	PSO-5	PSO-6	PSO-7	PSO-8
CO1	3	3	3	3	3	3	2	2
CO2	3	2	3	2	2	2	3	3
CO3	3	3	3	3	2	2	3	3
CO4	2	3	3	2	3	3	2	2

Paper – 711C: FRACTIONAL CALCULUS(4L-0T-0P)

Full Marks: 100 (20 Mid Term + 80 End Term)

Pre-requisites:
Ordinary differential equations, Partial differential equations

Objective:

Syllabus-:

Unit I **[12 Hours]**

Fractional derivatives: Gamma function and its properties, Beta function, Contour integral representation. Fractional derivatives: Grunwald-Letnikov, Riemann-Liouville and Caputo's fractional derivative, Leibniz rule for fractional derivatives, Geometric and physical interpretation of fractional integration and fractional differentiation.

Unit II **[12 Hours]**

Sequential fractional derivatives: Left and right fractional derivatives. Properties of fractional derivatives. Laplace transforms of fractional derivatives. Fourier transforms and Mellin transforms of fractional derivatives.

Unit III **[12 Hours]**

Linear Fractional Differential Equations: Fractional differential equation of a general form. Existence and uniqueness theorem as a method of solution. Dependence of a solution on initial conditions. The Laplace transform method. Standard fractional differential equations. Sequential fractional differential equations.

Unit IV **[12 Hours]**

Fractional Differential Equations: Introduction, linearly independent solutions, Solutions of the homogeneous and nonhomogeneous fractional differential equations, Reduction of fractional partial differential equations to ordinary differential equations.

Prescribed Textbooks:

1. Miller K.S. and Ross B. *An Introduction to the Fractional Calculus and Fractional Differential Equations*, John Wiley and Sons, New York, 1993.
2. Oldham K.B. and Spanier J. *The Fractional Calculus*, Academic Press Inc., USA, 1974.
3. Samko S.G., Kilbas A.A., Marichev O.I., *Fractional Integrals and Derivatives*, Gordon and Breach Science publishers, 1987.
4. Podlubny I., *Fractional Differential Equations*, Academic Press, 1998.

Course Outcomes

After the successful completion of this course the students will be able to-:

CO1: Demonstrate understanding of the basic concepts underlying the definition of the general Lebesgue measure in real line and properties.

CO2: Demonstrate understanding of the statements of the main results on integration on Real line and an ability to apply these in examples.

CO3: Apply the theory of the concept of differentiation to solve a variety of problems at an

Appropriate level of difficulty.

CO4: Demonstrate skills in communicating mathematics orally and in writing of abstract measure

CO-PO Mapping (Paper 711C)

	PO-1	PO-2	PO-3	PO-4	PO-5	PO-6	PO-7
CO1	3	2	1	1	2	1	1
CO2	3	1	3	2	2	1	2
CO3	2	3	3	3	2	2	2
CO4	2	2	3	2	3	2	3

Programme articulation matrix row for Paper 711C

	PO-1	PO-2	PO-3	PO-4	PO-5	PO-6	PO-7
Course Paper 711C	3	2	3	2	3	2	3

CO-PSO Mapping (Paper 711C)

	PSO-1	PSO-2	PSO-3	PSO-4	PSO-5	PSO-6	PSO-7	PSO-8
CO1	3	3	3	3	3	3	2	2
CO2	3	2	3	2	2	2	3	3
CO3	3	3	3	3	2	2	3	3
CO4	2	3	3	2	3	3	2	2

The syllabus of 712 will be same as given by UGC in December 2019. It will run centrally.

Paper – 713: RESEARCH METHODOLOGY (2L-0T-0P)

Full Marks: 100 (20 Mid Term + 80 End Term)

Syllabus.

Unit-I **[12 Hours]**

Scientific research and literature survey. History of mathematics, finding and solving research problems, role of a supervisor, survey of a research topic, publishing a paper, reviewing a paper, research grant proposal writing, copyright issues, ethics and plagiarism, Research tools. Searching google, MathSciNet, ZMATH, Scopus, ISI Web of Science, Impact factor, h-index, Google Scholar, ORCID, JStor, Online and open access journals, Virtual library of various countries. Nature of Intellectual Property: Patents, designs, trade and copyright, Process of patenting and development: technological research, innovation, patenting, development, procedure for grants of patents. Patent rights, licencing and transfer of Technology, Patent information and data base, geographical indications

Unit-II **[12 Hours]**

Quantitative Data Analysis: Types of Data, Data collection: Methods and tools, Hypothesis testing, Normal and binomial distribution and their property, Tests of significance: t-test, F-test, Chi-square test, Correlation and Regression.

Unit-III **[12 Hours]**

Scientific writing and presentation. Writing a research paper, survey article, thesis writing; Learning LaTeX, PSTricks, Presentation using Beamer, HTML and MathJaX. Introduction to MS-Office software, MS-word, MS-Excel, MS-power point, Preparation of Poster, Electronic submission of manuscripts, communication skills: oral and poster

Unit-IV **[12 Hours]**

Software for Mathematics. Mathematica/Matlab/Scilab/GAP

Reference:

1. J. Stillwell, Mathematics and its History, Springer International Edition, 4th Indian Reprint, 2005
2. L. Lamport, LaTeX, a Document Preparation System, 2nd ed, Addison-Wesley, 1994.
3. Norman E. Steenrod, Paul R. Halmos, Menahem M. Schiffer, Jean A. Dieudonne, How to Write Mathematics, American Mathematical Society, 1973.
4. Nicholas J. Higham, Handbook of Writing for the Mathematical Sciences, 2ed, SIAM, 1998.
5. Donald E. Knuth, Tracy L. Larrabee, and Paul M. Roberts, Mathematical Writing, Mathematical Association of America, Washington, D.C., 1989
6. Frank Mittelbach, Michel Goossens, Johannes Braams, David Carlisle, Chris Rowley, The LaTeX Companion, 2nd edition (TTCT series), Addison-Wesley, 2004.
7. Michel Goossens, Frank Mittelbach, Sebastian Rahtz, Denis Roegel, Herbert Voss, The LaTeX Graphics Companion, 2nd edition (TTCT series), Addison-Wesley, 2004
8. Martin J. Erickson and Donald Bindner, A Student's Guide to the Study, Practice, and Tools of Modern Mathematics, CRC Press, Boca Raton, FL, 2011.
9. Mathtools documentation (http://mirrors.ctan.org/macros/latex/contrib/mathtools/mathtools.pdf)
10. Pstricks documentation (http://tug.org/PSTricks/main.cgi?file=doc/docs)

11. MathJax documentation (http://tug.org/PSTricks/main.cgi?file=doc/docs)